Brunei In-Country Training: REALISTIC MATHEMATICS EDUCATION (RME) FOR MEANINGFUL LEARNING IN PRIMARY MATHEMATICS

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Introduction

This article highlights an approach to teaching and learning theory in mathematics education which promotes the importance of concept formation and problem solving. This approach known as Realistic Mathematics Education (RME) has been used successfully in many countries notably in the founding country, the Netherlands. The present form of RME outlined here is mostly determined by Professor Hans Freudenthal, its founding father, and his colleagues based on their views on mathematics education.

Background

Realistic Mathematics Education or in short RME is a theoretical approach toward mathematics teaching and learning. RME combines the idea of what is mathematics, how mathematics should be learnt, and how mathematics should be taught. These are the three pillars of RME. It was developed in Freudenthal Institute, which was established in 1971 under the University of Utrecht, the Netherlands. The institute was named after Professor Hans Freudenthal (1905 - 1990), a German/Dutch author, educator and mathematician. He believed that pupils should not be considered as passive receivers of ready-made mathematics. Instead, he advocated that education be arranged so as to allow pupils to use every opportunity and situation to reinvent mathematics themselves.

RME is the Dutch reaction to both the American’s New Math movement and to the then prevailing Dutch approach to mathematics education, which is labelled as mechanistic mathematics education. Although much development work has been carried out since its conception, development in RME is considered as ‘work under construction’. This is due to the fact that until now the struggle against mechanistic approach to mathematics has not been completely conquered. Also, RME which is based on the idea of mathematics as a human activity can never be considered a fixed and finished theory of mathematics education. Human is evolving therefore the human activity is ever changing. As a result so must mathematics and hence mathematics education.

The RME Approach

RME uses context problems to help pupils develop mathematically, emphasizing on pupils through making sense of the subject. Problems are developed from various contexts which are considered meaningful as learning resources. Context problems are used as both a starting point and the medium through which pupils develop understanding. This relates strongly to Freudenthal’s (1977) view that the mathematics we teach must be connected to reality, stay closed to children, and be relevant to society so as to be of human value. This will enable the pupils to make a start using their own common sense and prior knowledge.
Experience shows that, through staying connected with the context, pupils are able to continue to make sense of what they are doing. They do not need to resort to memorizing rules and procedures which make no sense to them.

However, a formal mathematical problem could be the starting point in the case of some pupils, if it is accessible to the learner. According to Van den Heuvel-Panhuizen (2001), the key word here is ‘realisable’ or ‘imaginable’ by the learner, rather than realistic. Hence, the word “realistic”, does not just refer to the connection with the real-world, but also refers to problem situations which are real in pupils’ mind. In Dutch language, the verb “to imagine” is “zich REALISERen”. It is this significance and emphasis on making something real in the minds of the pupils that gave RME its name. Also, de Lange (1996) elaborated that problem situations can also be seen as application or modelling and not always necessary be a real-world context.

Mathematical concepts are developed by mathematization process that is starting from context-link solution pupils gradually develop tools for mathematical understanding to formal level. According to de Lange (1996), “the process of developing mathematical ideas and concepts starting from the real world” is known as conceptual mathematization. In conceptual mathematisation, the pupils are forced to explore the situation presented by the context problem, identify the relevant mathematics, visualize and schematize, and develop a model resulting in a mathematical concept. The generalization and reflection by the pupils will develop a more complete concept. Then, they can apply these mathematical concepts to new areas of the real world. In this way, the pupils will be able to reinforce and strengthen the concept. The diagram in Figure 1, depicts this learning process.

![Conceptual Mathematisation Diagram](image)

Figure 1: Conceptual Mathematisation

Models which emerged in pupils’ mathematical activities might prompt interactivities that lead to a higher level of mathematical thinking. The initial realistic problem which makes sense should enable the pupil to have a mental image of the situation. During the course of the text, the pupils will be offered various models or ways of representing and working with the problem. At the informal level, these models could take the form of sketches, pictures, diagrams or long ways of recording a calculation. Pupils would be encouraged to work with their informal approaches and gradually refining these as appropriate. At a formal level, these models may be more abstract mathematical tools such as table, symbols for representation, formulae or more generalisable strategies.
Models bridge the gap between the informal and the formal. The models need to shift from a “model of” to a “model for”. This is a crucial mechanism for the growth of understanding. A “model of” is a context-based model, formed and developed initially from a problem situation. A “model of” has close relationship with the problem situation. When the “model of” is developed and generalized independently of the problem situation, it is then known as the “model for”. Models also permit pupils to work at differing levels of abstraction. Hence, those who have difficulty with the formal notions can still make progress and will still have strategies for solving problems. In RME, formalization is reached in several consecutive stages. It is important that pupils are expected to operate only at a level of formality which is accessible and understandable to them. The process whereby pupils start with their informal methods and, by reflection begins to represent their thinking in a more formal way is known as “progressive mathematisation” (Van den Heuvel-Panhuizen, 2003). It is an indicative of how pupils make progress within this approach.

Freudenthal (1991) also stressed the need to provide our pupils the “guided” opportunity to “re-invent” mathematics by letting them do it under guidance. The focus should not be on mathematics as a closed system but on the activity and on the process of mathematisation (Freudenthal, 1968). Mathematics education structured as a process of guided reinvention, allows the pupils to experience a similar process compared to the process by which mathematics was invented. According to him, “guided” refers to the instructional environment of the learning process while “invention” refers to the steps involved in the learning processes. Figure 2, shows the guided reinvention model.

![Guided Reinvention Model](image)

Figure 2: Guided Reinvention model (Gravenmeijer, 1994)

Treffers (1987), highlighted that there are two types of mathematisation formulated explicitly in an educational context. They are the horizontal and vertical mathematisation. Gravemeijer (1994) depicted these two types of mathematisation as reinvention process. In horizontal mathematisation, the pupils develop mathematical tools which can help to arrange systematically and solve a problem located in a realistic situation. They will describe the problems using their own language and symbols, and solve the problems. They will create their own strategies and these strategies might be different from others.
On the other hand, vertical mathematisation is the process of reorganization within the mathematical system itself. This could be for instance, finding shortcuts, representing a relation in a formula, proving regularities, refining and adjusting models, using different models, combining and integrating models formulating a mathematical model, discovering connections between concepts and strategies, and then applying these discoveries. Freudenthal (1991) also asserted that horizontal mathematisation involves going from the world of life into the world of symbols, while vertical mathematisation means moving within the world of symbols. However, he stressed that the differences between these two types are not always clear cut and both are of equal value. Furthermore, mathematisation can occur on different levels of understanding.

Classification of Mathematics Education

According to Treffers (1987), we can identify RME from the other theories of mathematics instruction using horizontal and vertical mathematisation. He proposed four different ways of categorizing mathematics education using these two types of mathematisation. The resultant four categories are described as mechanistic, empiristic, structuralist and realistic as shown in Table 1.

Table 1: Four types of mathematics education

<table>
<thead>
<tr>
<th>Type</th>
<th>Horizontal Mathematisation</th>
<th>Vertical Mathematisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanistic</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Empiristic</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Structuralist</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Realistic</td>
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</tbody>
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The mechanistic approach lacks both the horizontal and vertical mathematisation. Pupils are told what to do and how to do it and they practise procedures so as to reinforce them. It is based on drill-practice and patterns. This approach treats the pupil like a machine (mechanic).

In the empiristic approach which is rich in horizontal mathematisation but lacks vertical mathematisation, the world is the reality. The pupils are provided with materials from their living world. Normally, they are not prompted to come up with a formula or model for an extended situation. According to Treffers (1987), in general, this approach is seldom taught. It lacks the support of models, schemas and the like which make it difficult to arrive at the formal level.

In the structuralistic approach, it is rich in vertical mathematisation but lacks a horizontal component. In this approach, operations, mathematical patterns, and the like are demonstrated by using tools and media of instruction as representation of mathematical ideas and concepts. Vertical mathematisation is actualised by using those structured materials. However, the application of mathematics could not be realised unless the pupils have understood how to use learned procedures. This will result in the pupils not being able to develop further their natural and informal procedures.
The realistic approach is rich in the horizontal and vertical mathematisation. In this approach, the pupils are given the opportunity to explore the real data or pattern (in context). Then, they formalise it in a mathematical way and use the mathematical system derived to get new information or learn new mathematics.

The Five Tenets or Principles of RME

Historically, the characteristics of RME are related to the three Van Hiele’s levels for learning mathematics (de Lange, 1996). The process of learning mathematics is assumed to proceed through three levels:

(1) a pupil reaches the first level of thinking as soon as he/she can manipulate the known characteristics of a pattern that is familiar to him/her.

(2) a pupil reaches the second level the moment he/she learns to manipulate the interrelatedness of the characteristics.

(3) a pupil reaches the third level when he/she starts manipulating the intrinsic characteristics of relations.

The RME approach starts from the first level as in contrast to traditional instruction which is inclined to start at the second or third level. According to de Lange (1996), the skipping of the first level would not have happened if we start from real world application and modeling. The combination of the Van Hiele’s levels, Freudenthal’s didactical phenomenology and the progressive mathematisation result in the following five tenets or principles of RME:

(1) the use of contextual problems. It is important to use real contexts so as to make sense, meaningful, natural, and familiar to the pupils as the starting point so that they can immediately be engaged in the situation.

(2) the use of models or bridging by vertical instruments. Pupils develop and use models as a bridge between the abstract and real when solving problems. In the beginning, it is the model of situation that is familiar to the pupils. After undergoing through a process of generalizing and formalizing, the model eventually becomes an entity of its own. This is then used as a model for mathematical reasoning.

(3) the use of pupils own productions and constructions or pupils contribution. Opportunities should be provided to the pupils to produce more concrete things themselves and to develop their own informal problem solving strategies. Much contribution should come from the pupil’s own construction, which will lead them from their own informal to the more standard formal methods. The teacher and the instructional materials will guide a bottom-up reinvention process of the pupils.

(4) the interactive character of the teaching process or interactivity. In RME, the interaction among pupils and between pupils and teacher is an essential part in the constructive learning process because discussion and collaboration enhance reflection.
on the work (Gravemeijer, 1994; de Lange, 1995). Here the pupil’s informal strategies
are used as a level to attain the formal ones. In an interactive instruction, pupils are
engaged in explaining, justifying, negotiating, agreeing and disagreeing, discussing,
questioning alternatives, evaluating, and reflecting.

(5) the intertwining of various learning strands. In RME, the integration of
mathematical strands or units is essential and it is often known as holistic approach. It
incorporates applications and implies that the learning strands cannot be dealt as
separate entities. Instead, an intertwining of learning strands is exploited in problem
solving. One of the reasons is that applying mathematics is very difficult if mathematics
is taught “vertically”. Moreover, solving rich context problems often means that the
pupils have to apply a broad range of mathematical tools and understanding.

Learning Trajectories

The teacher’s role in mapping out a learning route for instructional tasks is an important
feature of the RME. This is realised by anticipating paths of development in pupils’
understanding and skills by drawing on knowledge of how conceptual structures develop within
a particular content domain and insights into children’s informal knowledge structures. Simon
(1995) describes the learning route as the hypothetical learning trajectory. The learning
trajectory is hypothetical because we can never be sure what they will do or whether and how
they will construct new interpretations, ideas, and strategies until they are really working on
the problem. Figure 3 depicts a hypothetical learning trajectory.

![Figure 3: Hypothetical Learning Trajectory (Simon, 1995)](image)

In 1997, the Freudenthal Institute and the SLO (the Dutch Institute for Curriculum
Development) started the TAL project (TAL is a Dutch abbreviation and stands for Intermediate
Goals Annex Learning-Teaching Trajectories) with the purpose to develop learning-teaching
trajectories for all the domains of the primary school mathematics curriculum. In total there
were three learning-teaching trajectories developed. They are a trajectory for whole number
calculation, one for measurement and geometry, and one for fractions, decimals and
percentages. They are intended to provide teachers with a mental educational map to help
them guide the pupils. The trajectory clarifies how abilities are built up in connection with each
other. It shows what is coming earlier and what is coming later. It does not only describes the
landmarks in pupils learning that can be recognised en route, but it also portrays the key activities in teaching that lead to these landmarks. It makes clear what is learned at one stage is understood and performed on a higher level in the following stage.

Iceberg Model

The Freudenthal Institute developed the iceberg model for use with teacher professional development to support teacher thinking about learning processes and strategies used by pupils. It is a visual model to distinguish the role of informal, pre-formal, and formal representations used by pupils. It is a powerful metaphor for illustrating how students need to experience a broad range of mathematical models to make sense of formal mathematical representations. The iceberg consists of the “tip of the iceberg” and a much larger area underneath, the “floating capacity”. The “tip of the iceberg” represents formal mathematical goals. The pre-formal representations are at the “water line,” and informal representations are at the bottom. The “floating capacity” suggests that underneath the formal mathematics exists an important network of representations that can be used to increase student access to and support student understanding of mathematics. Figure 4 depicts an Iceberg Model.

![Iceberg Model](image)

Figure 4: Iceberg model

RME Instructional Design Principles

There are three key heuristics principles of RME for the process of instructional design which were identified by Gravemeijer (1994, 1999). They are the guided reinvention through progressive mathematisation, didactical phenomenology, and self developed or emergent models.
The principle of *guided reinvention* requires that well-chosen contextual problems be presented to learners that offer them opportunities to develop informal, highly context-specific solution strategies (Doorman, 2001). These informal solution procedures may then function as foothold inventions for formalisation and generalisation, a process referred to as "progressive mathematising" (Gravemeijer, 1994). The reinvention process is set in motion when learners use their everyday language (informal description) to structure contextual problems into informal or more formal mathematical forms. The instructional designer therefore tries to compile a set of problems that can lead to a series of processes that together result in the reinvention of the intended mathematics. With guidance, the learners are afforded the opportunity to construct their own mathematical knowledge store on this basis.

The principle of didactical phonomenology implies that in learning mathematics, one has to start from phenomena meaningful to the learner, and that implore some sort of organizing be done and that stimulate learning processes. This principle should fulfill four functions:

- Concept formation (to allow learners natural and motivating access to mathematics),
- Model formation (to supply a firm basis for learning the formal operations, procedures, and rules in conjunction to other models as the support for thinking),
- Applicability (to utilise reality as a source and domain of applications),
- Practice (to exercise the specific abilities of learners in applied situations).

(Treffers and Goffree, 1985)

The third principle for instructional design in RME, the principle of emergent or self-developed models, plays an important role in bridging the gap between informal and formal knowledge. In order to achieve this, the learners need to be given opportunities to use and develop their own models when solving problems. Learners further enhance their former models and their knowledge about mathematics. In this way, the symbolisations that comprise the model and those rooted in the process of modelling can change over time. Learners therefore progress from what is termed a "model-of" a situated activity to a "model-for" more sophisticated reasoning.

**The Role of Context in RME**

In RME, a context plays a significant role. It is the context that distinguishes RME from the other mathematics teaching approaches, such as the mechanistic and structuralist approaches. When context problems are used, the instruction is directed to the process of reinvention mathematical concepts through both the horizontal and vertical mathematisation. Also, with the use of context problem as the starting point, teaching will engage pupils in meaningful mathematical activities and trigger interactivity among pupils.

According to de Lange (1987), the role of context in RME is twofold. Its first role is as the source of conceptual mathematisation and the other role is as a field of mathematical concepts. Figueiredo (1999), indicated that to allow pupils to engage in more meaningful context problems practices, the nature of contexts and how they need to be used must be different. For this reason, he highlighted that context in RME must have the following characteristics:
- Easy to imagine, easy to recognize and appealing
- Familiar to the pupils
- The problem itself can come to the fore out of the described situations
- Demand mathematical organization (progressive mathematis
tion)
- Not separated from the process of problem solving but it must lead pupils to arrive at a solution.

Hence, context problems in RME fulfil a number of functions. They are:

- Help pupils to understand the purpose of the problem quickly;
- Provide pupils with strategies that are based on their own experiences and informal knowledge;
- Offer pupils more opportunities to demonstrate their abilities;
- Motivate pupils to solve the problems.

Assessment

In RME, assessment is considered to be an integral component of the teaching and learning process. It is not an after-lesson activity which is normally practised in traditional classrooms. Hence, assessment based on RME includes doing the assessment during the lesson such as asking the pupils to write an essay, perform an experiment, collecting data and to design exercises that can be used in a test, or to design a test for other pupils in the classroom. Assessment too can be in the form of giving pupils problems as homework.

De Lange (1995) formulated the following five principles of assessments as a guide in performing assessment in RME:

- The primary purpose of testing is to improve learning and teaching. It means assessment should measure the pupils during the teaching-learning process in addition to end of unit or course.
- Methods of assessment should enable the pupils to demonstrate what they know rather that what they do not know. It can be conducted by having the problems that have multiple solutions with multiple strategies.
- Assessment should operationalise all the goals of mathematics education, lower, middle, and higher order thinking level.
- The quality of mathematics assessment is not determined by its accessibility to objective scoring. As such, objective tests and mechanical tests should be reduced. Instead, the pupils should be provided with tests that we really can see whether they understand the problems.
- The assessment tools should be practical, available to the applications in school cultures, and accessible to outside resources.

Another important component of assessment is the pupils’ (own) free productions. In addition, assessment in RME is context dependent. According to de Lange (1993), context-independent testing is deemed to be unfair. The context of assessment not only reflects the pupils’ real world, but also the “real world of mathematics itself” (de Lange, 1993). This implies
that assessment problems need to involve pupils’ abilities to think and communicate mathematically. They are to demonstrate higher order thinking skills associated with the discipline of mathematics. Heuvel-Panhuizen and Gravemeijer (1991) suggested that the best way to uncover what pupils are capable of doing is to elicit their own productions. This is done by asking pupils to think up problems. In this way, it does not only reveal what pupils are capable of doing but also their problem-solving processes. Offering pupils with situations in which they are allowed to work in their own ways have the possibility of enabling them to show creativity in their solutions, portray a problem-solving attitude, and reveal their full learning potential.

Conclusion
With the progressive development of mathematics and mathematics education, various schools of thoughts on what mathematics actually is, how best mathematics is actually learnt and how it should actually be taught most effectively naturally emerged. The Dutch RME presents a theoretical framework based on the understanding of what mathematics is and how our students learn it, and from this understanding determines how teachers should teach it. RME continually works toward the progress of pupils. The key RME principles of guided reinvention, didactical phenomenology and self-developed models give guidance in instructional design. Utilising these principles, models will originate from the context situations and function as bridges to higher levels of understanding. Finally, taken into consideration the Dutch achievement in the TIMSS, it seems that RME can elicit progress in mathematics achievements.
References


